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Penrose limit and string quantization in $AdS_3 \times S^3$

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Abstract: We consider corrections to the Penrose limit of $AdS_3 \times S^3$ with NS-NS flux which are due to the terms next to leading order in inverse radius expansion. The world-sheet theory of a lightcone string is interacting due to the presence of quartic terms in the action. Perturbative corrections to the spectrum are shown to agree with the results from the exact quantization in $AdS_3 \times S^3$.

Keywords: Bosonic Strings, Penrose limit and pp-wave background.

Contents

1. Introduction

String theory in the pp wave backgrounds has received a lot of attention recently (see e.g. [1]–[4]). These backgrounds are among the few ones with non-zero RR fields where string theory is exactly solvable. pp waves can also be obtained as the Penrose limit of $AdS_p \times S^q$ spaces, which have holographic descriptions. In a recent paper [1] Berenstein, Maldacena and Nastase (BMN) proposed that the AdS/CFT correspondence [5, 6, 7] which relates string theory in $AdS_5 \times S^5$ to four dimensional $\mathcal{N} = 4$ Super Yang-Mills, can be used to obtain the holographic dual of string theory in the pp wave background. The important result of their paper is the construction of boundary operators that correspond to excited string states in the bulk. In order to have finite lightcone momentum in the pp wave background, these states must have spacetime energy and spin which go to infinity, as the Penrose limit is taken. Corresponding operators are shown to have perturbative (in Yang-Mills coupling) corrections to the lightcone energy which sum up to reproduce exactly the spectrum of a string propagating in the pp wave background. It is interesting to study analogs of BMN construction for other backgrounds which have holographic duals. Among such theories, the near horizon limit of D1/D5 system [8, 9] takes a special place. By using S-duality this system can be mapped [10] to the near horizon limit of k NS5 branes and p fundamental strings [11, 12, 13]. The resulting closed string theory on $AdS_3 \times S^3 \times T^4$ is exactly solvable when the string coupling is small, which happens when $p \gg k$. It has been shown [14, 15] that the boundary superconformal algebra can be constructed from the worldsheet point of view. However the details of the boundary theory are not completely understood. (See e.g. [16]–[19] for some comments on this issue).

Although one is yet to achieve better understanding of D1/D5 boundary theory which may be needed for the construction of an analog of BMN correspondence, it is still possible to relate the theory in the pp wave background, which arises as the Penrose limit of $AdS_3 \times S^3 \times T^4$, to the exactly solvable theory in the original space. It therefore seems natural to take this example as a playground for studying various aspects of the Penrose limit in string theory. In this paper we simplify the model even further and consider the bosonic string on $AdS_3 \times S^3 \times \mathcal{M}$ where $\mathcal{M}$ is a unitary CFT of central charge 20, which is needed to make the string theory critical. We exhibit physical states in this background which flow into string states in the corresponding pp wave background as the Penrose limit is taken. An analogous correspondence in $AdS_5 \times S^5$ has been investigated recently from the sigma model point of view in refs. [20, 21]. We also consider next to leading order corrections to the pp wave background and write down the interacting worldsheet theory of a lightcone string. Perturbative corrections to the lightcone string spectrum are shown to precisely match the results from the string quantization in $AdS_3 \times S^3 \times \mathcal{M}$.

This paper is organized as follows. In the next section we study the lightcone string in the Penrose limit of $AdS_3 \times S^3 \times \mathcal{M}$ in the leading order in $1/k$ expansion, reproducing the results of [1, 4, 22]. In section 3 we consider next to leading order corrections to the spectrum. In section 4 we show that the spectrum is reproduced from the exact quantization of a string in $AdS_3 \times S^3 \times \mathcal{M}$. In section 5 we discuss our results and mention some directions for future work.

2. String in the Penrose limit of $AdS_3 \times S^3$ in the leading order

Let us briefly review bosonic string theory on $AdS_3 \times S^3 \times \mathcal{M}$ with the NS-NS flux. (For a more complete discussion see e.g. [14, 15].) $\mathcal{M}$ here denotes a unitary CFT with the central charge 20. The simplest example is just T^{20}. In the rest of the paper we will consider $\mathcal{M}$ to be in its ground state, although the incorporation of excitations is straightforward. The metric on $AdS_3 \times S^3$ is given by

$$ds^2 = k\left(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2 + \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\chi^2\right), \qquad (2.1)$$

Here and in the rest of the paper we set $\alpha' = 1$. The NS-NS flux is proportional to the volume form

$$H = -2k \sinh 2\rho \, dt \wedge d\rho \wedge d\phi - 2k \sin 2\theta \, d\psi \wedge d\theta \wedge d\chi \,. \qquad (2.2)$$

String theory in this background is described by a product of bosonic SL(2) and SU(2) WZNW models at levels $k_{\rm SL} = k+2$ and $k_{\rm SU} = k-2$ respectively. The $\mathrm{SL}(2)_L \times \mathrm{SL}(2)_R$ and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ Kac-Moody algebras are generated by the holomorphic and antiholomorphic currents. The mode expansion of holomorphic currents is given by

$$J^{a,\rm SL}(z) = \sum_{n=-\infty}^{\infty} \frac{J_n^{a,\rm SL}}{z^{n+1}}\,, \qquad a = \pm, 3, \qquad (2.3)$$

$$J^{a,\rm SU}(z) = \sum_{n=-\infty}^{\infty} \frac{J_n^{a,\rm SU}}{z^{n+1}}\,, \qquad a = \pm, 3, \qquad (2.4)$$

and the expression for antiholomorphic currents is similar. The central charge of the full theory is

$$c = \frac{3k_{\rm SL}}{k_{\rm SL}-2} + \frac{3k_{\rm SU}}{k_{\rm SU}+2} + 20 = 26\,. \qquad (2.5)$$

The states fall into representations of the algebra with the given SL(2) and SU(2) spins h and j and $J_0^{3,\mathrm{SL}}, \tilde{J}_0^{3,\mathrm{SL}}, J_0^{3,\mathrm{SU}}, \tilde{J}_0^{3,\mathrm{SU}}$ quantum numbers $m_{\mathrm{SL}}, \tilde{m}_{\mathrm{SL}}, m_{\mathrm{SU}}, \tilde{m}_{\mathrm{SU}}$. The BRST invariant states must satisfy physical state conditions which will be studied in section 4

The Penrose limit [23, 24] of (2.1) corresponds to the metric that is seen by a highly boosted particle. It is defined by rescaling the coordinates

$$X^+ = \frac{(t+\psi)}{2}, \qquad X^- = k\frac{(t-\psi)}{2}, \tag{2.6}$$

$$\rho = \frac{r}{\sqrt{k}}, \qquad \theta = \frac{y}{\sqrt{k}}, \tag{2.7}$$

and taking the limit $k \to \infty$. This transforms $AdS_3 \times S^3$ metric into

$$\begin{aligned} ds_0^2 &= -4dX^+dX^- + ds^2_{\mathrm{AdS},0} + ds^2_{\mathrm{S},0}\,, &(2.8)\\ ds^2_{\mathrm{AdS},0} &= -r^2(dX^+)^2 + dr^2 + r^2 d\phi^2\,, &(2.9)\\ ds^2_{\mathrm{S},0} &= -y^2(dX^+)^2 + dy^2 + y^2 d\chi^2 = -|Z|^2(dX^+)^2 + |dZ|^2\,, &(2.10)\end{aligned}$$

where $ds^2_{\mathrm{AdS},0}$ and $ds^2_{\mathrm{S},0}$ depend on AdS_3 and S^3 coordinates respectively. In eq. (2.10) we introduced a complex coordinate $Z = ye^{i\chi}$ for the S^3 part of the metric. The H field in the leading order is given by

$$\begin{aligned} H_0 &= H_{\mathrm{AdS},0} + H_{\mathrm{S},0}\,, &(2.11)\\ H_{\mathrm{AdS},0} &= -4r\, dX^+ \wedge dr \wedge d\phi\,, &(2.12)\\ H_{\mathrm{S},0} &= -4y\, dX^+ \wedge dy \wedge d\chi = -2i\, dX^+ \wedge dZ \wedge d\bar{Z}\,. &(2.13)\end{aligned}$$

Note that in the leading order the AdS_3 metric and NS-NS flux are the same as their S^3 counterparts. Therefore it is sufficient to consider the S^3 part. The incorporation of $\mathcal{M}$ is not difficult, as it does not scale under the Penrose limit.

Let us start by doing the lightcone quantization in the leading order in $1/k$ expansion. The background to this order is described by the metric (2.10) and the B field

$$B = -idX^+ \wedge (\bar{Z}dZ - Zd\bar{Z})\,. \tag{2.14}$$

We will be following closely the treatment of [25]. Let us denote the worldsheet coordinates by $\tau \in (-\infty, \infty)$ and $\sigma \in [0, l]$. The lightcone gauge is specified by setting

$$\begin{aligned} X^+ &= \tau\,, &(2.15)\\ \partial_\sigma \gamma_{\sigma\sigma} &= 0\,, &(2.16)\\ \det \gamma_{\alpha\beta} &= -1\,, &(2.17)\end{aligned}$$

where

$$\gamma^{\alpha\beta} = \begin{pmatrix} -\gamma_{\sigma\sigma}(\tau) & \gamma_{\sigma\tau}(\tau,\sigma) \\ \gamma_{\sigma\tau}(\tau,\sigma) & \gamma_{\sigma\sigma}^{-1}(\tau)(1-\gamma_{\sigma\tau}^2(\tau,\sigma)) \end{pmatrix} \tag{2.18}$$

is the worldsheet metric. The lagrangian takes the form

$$\begin{aligned} L_0 = -\frac{1}{4\pi}\int_0^l d\sigma\Big[&\gamma_{\sigma\sigma}\left(4\dot{x}^- - |\dot{Z}|^2 + |Z|^2\right) - 2\gamma_{\sigma\tau}\left(2(Y^-)' - \frac{1}{2}\left[\dot{Z}\bar{Z}' + \dot{\bar{Z}}Z'\right]\right) + \\ &+ \gamma_{\sigma\sigma}^{-1}(1-\gamma_{\sigma\tau}^2)|Z'|^2 - i(\bar{Z}Z' - Z\bar{Z}')\Big]\,, \end{aligned} \tag{2.19}$$

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where dots and primes denote ∂_τ and ∂_σ respectively, and x^- denotes the zero mode of X^-

$$x^-(\tau) = \frac{1}{l}\int_0^l d\sigma X^-(\tau,\sigma)\,, \tag{2.20}$$

and

$$Y^- = X^- - x^-\,. \tag{2.21}$$

As explained in [25], the equation of motion obtained by the variation of the action with respect to Y^-, together with the leftover gauge freedom $\sigma \to \sigma + f(\tau)$, allows one to set $\gamma_{\sigma\tau} = 0$. One can also treat $\gamma_{\sigma\tau}$ as a Lagrange multiplier. Integrating it out relates Y^- and $Z, \bar{Z}$

$$2(Y^-)' = \frac{1}{2}\left[\dot{Z}\bar{Z}' + \dot{\bar{Z}}Z'\right]. \tag{2.22}$$

One must supplement this equation with the constraint which ensures the single valuedness of Y^-

$$\int_0^l d\sigma\,(Y^-)' = 0\,. \tag{2.23}$$

The lagrangian becomes

$$L_0 = \int_0^l d\sigma \frac{1}{4\pi\gamma_{\sigma\sigma}}\left[-4\gamma_{\sigma\sigma}^2\dot{x}^- + \gamma_{\sigma\sigma}^2|\dot{Z}|^2 - |Z' + i\gamma_{\sigma\sigma}Z|^2\right]. \tag{2.24}$$

Since x^- does not explicitly enter the lagrangian, the canonical momentum conjugate to it is conserved and is related to $\gamma_{\sigma\sigma}$ as

$$p_- = -\frac{4\gamma_{\sigma\sigma}l}{4\pi}\,. \tag{2.25}$$

Let us define

$$\eta \equiv -\frac{p_-}{2}\,. \tag{2.26}$$

It will be convenient to choose the gauge

$$l = 2\pi\eta = -\pi p_-\,. \tag{2.27}$$

In the leading order in $1/k$ expansion $\eta = p^+$ and $\gamma_{\sigma\sigma} = 1$, but both relations change when next to leading order corrections are taken into account. It is convenient to make a change of variables

$$Z = e^{-i\sigma}W\,, \tag{2.28}$$

which introduces a complex field W satisfying

$$W(\sigma + l) = e^{2\pi i\eta}W(\sigma)\,. \tag{2.29}$$

Note that this equation is invariant under $p_- \to p_- + 2$. However we are going to be interested in the states which have $p_- \in [0,2]$. We will see that states of this type correspond to unitary representations of the SL(2) $\times$ SU(2) current algebra, which are constrained by $1/2 < h < (k+1)/2$. Other unitary representations, with h out of this bound, are obtained by the action of the spectral flow [26].

The lagrangian in terms of W becomes that of a free string. The hamiltonian then becomes

$$H_{lc,0} = \frac{1}{2}\int_0^l d\sigma \left[(8\pi)|\Pi_W|^2 + \frac{1}{2\pi}|W'|^2\right], \tag{2.30}$$

where

$$\Pi_W = \frac{1}{4\pi}\dot{\bar{W}} \tag{2.31}$$

The equations of motion for $W, \bar{W}$ are conventional wave equations, so the fields can be written as a sum of left and right moving modes

$$W = W_L(\tau+\sigma) + W_R(\tau-\sigma)\,, \tag{2.32}$$
$$\bar{W} = \bar{W}_L(\tau+\sigma) + \bar{W}_R(\tau-\sigma)\,, \tag{2.33}$$

which satisfy

$$\dot{W}_L = W_L'\,, \tag{2.34}$$
$$\dot{W}_R = -W_R'\,, \tag{2.35}$$

and have the following oscillator expansion

$$W_L = \frac{i}{\sqrt{2}}\sum \frac{\alpha_{n-\eta}}{n-\eta} e^{-\frac{i(n-\eta)(\tau+\sigma)}{\eta}}, \qquad W_R = \frac{i}{\sqrt{2}}\sum \frac{\tilde{\alpha}_{n+\eta}}{n+\eta} e^{-\frac{i(n+\eta)(\tau-\sigma)}{\eta}}, \tag{2.36}$$
$$\bar{W}_L = \frac{i}{\sqrt{2}}\sum \frac{\bar{\alpha}_{n+\eta}}{n+\eta} e^{-\frac{i(n+\eta)(\tau+\sigma)}{\eta}}, \qquad \bar{W}_R = \frac{i}{\sqrt{2}}\sum \frac{\tilde{\bar{\alpha}}_{n-\eta}}{n-\eta} e^{-\frac{i(n-\eta)(\tau-\sigma)}{\eta}}. \tag{2.37}$$

The corresponding expressions for canonical momenta are

$$\Pi_W = \frac{1}{4\sqrt{2}\pi\eta}\sum\left[\bar{\alpha}_{n+\eta} e^{-\frac{i(n+\eta)(\tau+\sigma)}{\eta}} + \tilde{\bar{\alpha}}_{n-\eta} e^{-\frac{i(n-\eta)(\tau-\sigma)}{\eta}}\right], \tag{2.38}$$
$$\Pi_{\bar{W}} = \frac{1}{4\sqrt{2}\pi\eta}\sum\left[\alpha_{n-\eta} e^{-\frac{i(n-\eta)(\tau+\sigma)}{\eta}} + \tilde{\alpha}_{n+\eta} e^{-\frac{i(n+\eta)(\tau-\sigma)}{\eta}}\right]. \tag{2.39}$$

The canonical commutation relations lead to the following relation for the oscillator modes[1]

$$[\alpha_{n-\eta}, \bar{\alpha}_{-m+\eta}] = 2(n-\eta)\delta_{n,m}\,, \tag{2.40}$$

$$[\tilde{\bar{\alpha}}_{n-\eta}, \tilde{\alpha}_{-m+\eta}] = 2(n+\eta)\delta_{n,m}\,. \tag{2.41}$$

It will be convenient to introduce canonically normalized creation and annihilation operators

$$a_{n+}^\dagger = \frac{\alpha_{-n-\eta}}{\sqrt{2(n+\eta)}}, \qquad a_{n+} = \frac{\bar{\alpha}_{n+\eta}}{\sqrt{2(n+\eta)}}, \qquad n>0 \tag{2.42}$$
$$a_{n-}^\dagger = \frac{\bar{\alpha}_{-n+\eta}}{\sqrt{2(n-\eta)}}, \qquad a_{n-} = \frac{\alpha_{n-\eta}}{\sqrt{2(n-\eta)}}, \qquad n>0 \tag{2.43}$$
$$a_0^\dagger = \frac{\alpha_{-\eta}}{\sqrt{2\eta}}, \qquad a_0 = \frac{\bar{\alpha}_\eta}{\sqrt{2\eta}}, \tag{2.44}$$

[1] Note that similar commutation relations appear for the string in the twisted sector of the orbifold [27].

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$$\tilde{a}^\dagger_{n-} = \frac{\tilde{\tilde{\alpha}}_{-n-\eta}}{\sqrt{2(n+\eta)}}\,, \qquad \tilde{a}_{n-} = \frac{\tilde{\alpha}_{n+\eta}}{\sqrt{2(n+\eta)}}\,, \qquad n > 0 \tag{2.45}$$

$$\tilde{a}^\dagger_{n+} = \frac{\tilde{\alpha}_{-n+\eta}}{\sqrt{2(n-\eta)}}\,, \qquad \tilde{a}_{n+} = \frac{\tilde{\tilde{\alpha}}_{n-\eta}}{\sqrt{2(n-\eta)}}\,, \qquad n > 0 \tag{2.46}$$

$$\tilde{a}^\dagger_0 = \frac{\tilde{\tilde{\alpha}}_{-\eta}}{\sqrt{2\eta}}\,, \qquad \tilde{a}_0 = \frac{\tilde{\alpha}_\eta}{\sqrt{2\eta}}\,. \tag{2.47}$$

Pluses and minuses in subscript denote the transformation properties of the oscillators under the rotation of complex plane $Z \to e^{i\theta}Z$. Substituting the oscillator expansions into (2.30) and using the commutation relations one can obtain the following form for the lightcone hamiltonian

$$H_{lc,0} = 2 + N' + \tilde{N}' + \frac{N + \tilde{N} - 2}{\eta} = 2 + N' + \tilde{N}' - 2\frac{N + \tilde{N} - 2}{p_-} \tag{2.48}$$

$$N = \sum_{n>0} n\left[a^\dagger_{n+}a_{n+} + a^\dagger_{n-}a_{n-}\right], \tag{2.49}$$

$$\tilde{N} = \sum_{n>0} n\left[\tilde{a}^\dagger_{n+}\tilde{a}_{n+} + \tilde{a}^\dagger_{n-}\tilde{a}_{n-}\right], \tag{2.50}$$

$$N' = a^\dagger_0 a_0 + \sum_{n>0}\left[a^\dagger_{n+}a_{n+} - a^\dagger_{n-}a_{n-}\right], \tag{2.51}$$

$$\tilde{N}' = \tilde{a}^\dagger_0\tilde{a}_0 + \sum_{n>0}\left[\tilde{a}^\dagger_{n-}\tilde{a}_{n-} - \tilde{a}^\dagger_{n+}\tilde{a}_{n+}\right]. \tag{2.52}$$

Additional factors of 2 arise due to normal ordering of the oscillators.[2] The hamiltonian (2.48) must be supplemented by a usual physical state condition,

$$N = \tilde{N}\,, \tag{2.53}$$

which is a consequence of (2.23). Note that the hamiltonian (2.48) contains a tachyonic state at level 0, but is nonnegative for states with $N > 0$ and $p_- \in [0, 2]$.

3. Next to leading order corrections to the Penrose limit

Our next task will be computing $\mathcal{O}(1/k)$ corrections to the lightcone spectrum (2.48). The corrections to the S^3 metric (2.10) and B field (2.14) are

$$\begin{aligned} ds^2_{\mathrm{S},1} &= \frac{1}{k}\left[2y^2 dX^+dX^- + \frac{y^4}{3}(dX^+)^2 - \frac{y^4}{3}(d\chi)^2\right] = \\ &= \frac{1}{k}\left[2|Z|^2 dX^+dX^- + \frac{|Z|^4}{3}(dX^+)^2 + \frac{1}{12}(Zd\bar{Z} - \bar{Z}dZ)^2\right], \end{aligned} \tag{3.1}$$

$$B_1 = \frac{1}{k}\left[i\frac{|Z^2|}{3}dX^+ \wedge (\bar{Z}dZ - Zd\bar{Z}) + idX^- \wedge (\bar{Z}dZ - Zd\bar{Z})\right]. \tag{3.2}$$

[2] In computing normal ordering constants one should keep in mind that the total central charge of the matter theory is 26 and $\mathcal{M}$ is in its ground state.

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Note that AdS_3 metric and B field can be obtained from their S^3 counterparts by the substitution $k \to -k$. Therefore in the calculations below we will only retain the S^3 part.

The correction to the lagrangian therefore becomes

$$L_1 = -\frac{1}{4\pi k}\int_0^l d\sigma \Big[-2|Z|^2\dot{X}^- - \frac{1}{3}|Z|^4 - \frac{1}{12}(Z\dot{\bar{Z}} - \bar{Z}\dot{Z})^2 + \frac{1}{12}(Z\bar{Z}' - \bar{Z}Z')^2 + \\ + i\frac{|Z|^2}{3}\left(\bar{Z}Z' - Z\bar{Z}'\right) + i\dot{X}^-\left(\bar{Z}Z' - Z\bar{Z}'\right) - i(X^-)'\left(\bar{Z}\dot{Z} - Z\dot{\bar{Z}}\right)\Big]. \tag{3.3}$$

Note that in the leading order in $1/k$ $\gamma_{\sigma\tau}$ is set to zero by the variation of Y^-. On the other hand, varying $\gamma_{\sigma\tau}$ implies that

$$\left(2(Y^-)' - \frac{1}{2}\left[\dot{Z}\bar{Z}' + \dot{\bar{Z}}Z'\right]\right) \sim \mathcal{O}\left(\frac{1}{k}\right). \tag{3.4}$$

Therefore we do not write terms containing $\gamma_{\sigma\tau}$, as they are higher order in $1/k$.

It will be convenient to separate the lagrangian (3.3) into three terms

$$L_1 = L_1^{(1)} + L_1^{(2)} + L_1^{(3)}, \tag{3.5}$$

$$L_1^{(1)} = -\frac{1}{4\pi k}\int_0^l d\sigma\left[-2|Z|^2\dot{x}^- + i\dot{x}^-\left(\bar{Z}Z' - Z\bar{Z}'\right)\right], \tag{3.6}$$

$$L_1^{(2)} = -\frac{1}{4\pi k}\int_0^l d\sigma\Big[-\frac{1}{3}|Z|^4 - \frac{1}{12}(Z\dot{\bar{Z}} - \bar{Z}\dot{Z})^2 + \\ + \frac{1}{12}(\bar{Z}Z' - Z\bar{Z}')^2 + i\frac{|Z|^2}{3}\left(\bar{Z}Z' - Z\bar{Z}'\right)\Big], \tag{3.7}$$

$$L_1^{(3)} = -\frac{1}{4\pi k}\int_0^l d\sigma\left[-2|Z|^2\dot{Y}^- + i\dot{Y}^-\left(\bar{Z}Z' - Z\bar{Z}'\right) - i(Y^-)'\left(\bar{Z}\dot{Z} - Z\dot{\bar{Z}}\right)\right]. \tag{3.8}$$

Let us analyze this expression, term by term. We first consider the effect of the addition of

$$L_1^{(1)} = \frac{1}{2\pi k}\int_0^l d\sigma\, \dot{x}^-\, \mathrm{Im}(\bar{W}W') \tag{3.9}$$

to the leading order lagrangian. The eq. (2.25) is modified to

$$p_- = -2\eta = -\frac{4\gamma_{\sigma\sigma} l}{4\pi} + \frac{1}{2\pi k}\int_0^l d\sigma\ \mathrm{Im}(\bar{W}_L W_L' + \bar{W}_R W_R' + \bar{W}_L W_R' + \bar{W}_R W_L'). \tag{3.10}$$

With the gauge choice (2.27) this implies

$$\gamma_{\sigma\sigma} = 1 + \delta\gamma_{\sigma\sigma}, \tag{3.11}$$

$$\delta\gamma_{\sigma\sigma} = \frac{N' + \tilde{N}'}{2k\eta} + \frac{1}{4\pi k\eta}\int_0^l d\sigma\ \mathrm{Im}(\bar{W}_L W_R' + \bar{W}_R W_L'), \tag{3.12}$$

where we used the oscillator expansion of fields to compute the integral

$$\int_0^l \frac{d\sigma}{2\pi}\ \mathrm{Im}(\bar{W}_L W_L' + \bar{W}_R W_R') = N' + \tilde{N}'. \tag{3.13}$$

In writing this result we omitted a constant which arises due to normal ordering of the zero modes. However when the AdS^3 part is added its zero modes come with a relative minus sign and therefore in the full $AdS_3 \times S^3$ expression the normal ordering constants will cancel.

At this point we would like to make some remarks on how perturbation theory works. Suppose one adds a small perturbation to the harmonic oscillator lagrangian

$$L = \frac{\dot{x}^2}{2} - \frac{w^2 x^2}{2} + \alpha f\,, \tag{3.14}$$

where $f = f(x, \dot{x})$ and α is a small parameter. The canonical momentum becomes

$$p = \dot{x} + \alpha \frac{\partial f}{\partial \dot{x}}\,. \tag{3.15}$$

We can write the new hamiltonian omitting terms $\mathcal{O}(\alpha^2)$.

$$H = \frac{p^2}{2} + \frac{w^2 x^2}{2} - \alpha f\,. \tag{3.16}$$

Hence to compute the change in the hamiltonian up to next to leading order due to $\mathcal{O}(1/k)$ corrections to the lagrangian we just need to subtract these terms from the unperturbed hamiltonian. Since the perturbation is already $\mathcal{O}(1/k)$, we can use the original oscillator expansions for the fields. The standard quantum mechanical perturbation theory then implies that the correction to the lightcone energy of a given string state is given by the diagonal matrix element. As explained above, the effect of $L_1^{(1)}$ is to change $\gamma_{\sigma\sigma}$ by a term $\mathcal{O}(1/k)$. This induces the $\mathcal{O}(1/k)$ correction to L_0, which we denote by δL_0. The corresponding correction to the lightcone energy is given by

$$\begin{aligned} H^{(1)}_{lc,1} = -\langle \delta L_0 \rangle &= \left\langle \frac{\delta\gamma_{\sigma\sigma}}{4\pi} \int_0^l d\sigma \left[-\dot{\bar{W}}\dot{W} - \bar{W}'W' + 2\,\mathrm{Im}(\dot{\bar{W}}W') \right] \right\rangle = \\ &= \left\langle \delta\gamma_{\sigma\sigma} \left(-\frac{(N+\tilde{N}-2)}{\eta} + \int_0^l \frac{d\sigma}{2\pi}\, \mathrm{Im}(\bar{W}_L W_R' + \bar{W}_R W_L') \right) \right\rangle. \end{aligned} \tag{3.17}$$

In obtaining this formula we expressed δL_0 in terms of $W, \bar{W}$ and used the oscillator expansions. Substituting (3.12) and using the fact that the expectation value is taken with respect to the eigenstate of oscillator number operators, we can rewrite (3.17) as

$$H^{(1)}_{lc,1} = -\frac{(N+\tilde{N}-2)(N'+\tilde{N}')}{2k\eta^2} + \frac{1}{2\eta k} \left\langle \left(\int_0^l \frac{d\sigma}{\pi}\, \mathrm{Im}[W_L \bar{W}_R'] \right)^2 \right\rangle, \tag{3.18}$$

where we used

$$\int_0^l d\sigma\ \mathrm{Im}(W_L \bar{W}_R' - W_L' \bar{W}_R) = \int_0^l d\sigma\, (\mathrm{Im}(2W_L \bar{W}_R' - (W_L \bar{W}_R)') = \int_0^l d\sigma\ \mathrm{Im}(2W_L \bar{W}_R')\,. \tag{3.19}$$

It is easy to compute the contribution of $L^{(2)}$ to the lightcone energy

$$H^{(2)}_{lc,1} = -\langle L^{(2)} \rangle = \left\langle \frac{1}{4\pi k} \int_0^l d\sigma\, \frac{1}{12} \left[(\bar{W}W' - W\bar{W}')^2 - (\bar{W}\dot{W} - W\dot{\bar{W}})^2 \right] \right\rangle. \tag{3.20}$$

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Substituting the explicit form of $L^{(2)}$, using the free field solution for $W, \bar{W}$ and taking advantage of the fact that we need to compute an expectation value, we have

$$H^{(2)}_{lc,1} = \left\langle \frac{1}{2\pi k} \int_0^l d\sigma \, W_L \bar{W}'_L W_R \bar{W}'_R \right\rangle . \tag{3.21}$$

The change in the light cone energy due to $L^{(3)}_1$ term in the lagrangian is

$$\begin{aligned} H^{(3)}_{lc,1} = -\langle L^{(3)} \rangle = \left\langle \frac{1}{2\pi k} \int_0^l d\sigma \right. & \left[-|W|^2 \dot{Y}^- + \mathrm{Im}(\dot{W}\bar{W})(Y^-)' - \right. \\ & \left. \left. - \mathrm{Im}(\bar{W}(-iW + W'))\dot{Y}^- \right] \right\rangle = \\ = \left\langle \frac{1}{2\pi k} \int_0^l d\sigma \right. & \left. \left[\mathrm{Im}(\dot{W}\bar{W})(Y^-)' - \mathrm{Im}(W'\bar{W})\dot{Y}^- \right] \right\rangle . \end{aligned} \tag{3.22}$$

In order to compute this correction we shall use (2.22) to express $(Y^-)'$ and $\dot{Y}^-$ in terms of oscillator modes. The eq. (2.22) can be written as

$$2(Y^-)' = Re \left[\dot{W} \left(i\bar{W} + \bar{W}' \right) \right] . \tag{3.23}$$

It will be convenient to express $W, \bar{W}$ in terms of left and right moving modes $W_L, \bar{W}_L$ and $W_R, \bar{W}_R$. Then (3.23) becomes

$$2(Y^-)' = -\mathrm{Im}(W'_L \bar{W}_L) + W'_L \bar{W}'_L + \mathrm{Im}(W'_R \bar{W}_R) - W'_R \bar{W}'_R - \mathrm{Im}(W_L \bar{W}_R)' . \tag{3.24}$$

We see that $(Y^-)'$ can be written as

$$(Y^-)' = (Y^-)'_L + (Y^-)'_R + (Y^-)'_{LR} , \tag{3.25}$$

where

$$(Y^-)'_L = \frac{1}{2} \left(-\mathrm{Im}(W'_L \bar{W}_L) + W'_L \bar{W}'_L \right) , \tag{3.26}$$

$$(Y^-)'_R = \frac{1}{2} \left(\mathrm{Im}(W'_R \bar{W}_R) - W'_R \bar{W}'_R \right) , \tag{3.27}$$

$$(Y^-)'_{LR} = -\frac{1}{2} \partial_\sigma \, \mathrm{Im} \left(W_L \bar{W}_R \right) . \tag{3.28}$$

Now one can easily find

$$\dot{Y}^- = (Y^-)'_L - (Y^-)'_R - \frac{1}{2} \partial_\tau \, \mathrm{Im}(W_L \bar{W}_R) + a(\tau) , \tag{3.29}$$

where $a(\tau)$ should be chosen such that

$$\int_0^l d\sigma \, \dot{Y}^- = 0 , \tag{3.30}$$

which is a consequence of the definition of Y^-,

$$\int_0^l d\sigma \, Y^- = 0 . \tag{3.31}$$

One can write down an explicit formula for $a(\tau)$,

$$a(\tau) = -\frac{1}{l}\int_0^l d\sigma\,\left((Y^-)'_L - (Y^-)'_R - \frac{1}{2}\,\mathrm{Im}(W'_L\bar{W}_R - W_L\bar{W}'_R)\right) = \qquad (3.32)$$
$$= -\frac{(N+\tilde{N}-2)}{2\eta^2} - \frac{1}{2l}\int_0^l d\sigma\,\mathrm{Im}(W_L\bar{W}'_R - W'_L\bar{W}_R)\,.$$

Now we are ready to compute the change in the light cone energy (we are omitting some terms whose expectation value is vanishing)

$$\begin{aligned} H^{(3)}_{lc,1} = &-\Big\langle \frac{1}{4\pi k}\int_0^l d\sigma\,\Big[2\left[W'_L\bar{W}'_L - \mathrm{Im}(W'_L\bar{W}_L)\right]\mathrm{Im}(W'_R\bar{W}_R) + \\ &+2\left[W'_R\bar{W}'_R - \mathrm{Im}(W'_L\bar{W}_R)\right]\mathrm{Im}(W'_R\bar{W}_R) - 4\,\mathrm{Im}(\bar{W}_RW'_L)\,\mathrm{Im}(\bar{W}_LW'_R)\Big]\Big\rangle - \\ &-\Big\langle\left(\frac{N+\tilde{N}-2}{\eta^2} + \frac{1}{l}\int_0^l d\sigma\,\mathrm{Im}(W_L\bar{W}'_R - W'_L\bar{W}_R)\right)\times \\ &\times\frac{1}{4\pi k}\int_0^l d\sigma\,\mathrm{Im}(W'_L\bar{W}_L + W'_R\bar{W}_R + \bar{W}_LW'_R + \bar{W}_RW'_L)\Big\rangle\,. \end{aligned} \qquad (3.33)$$

After some algebra one can arrive to the following expression

$$\begin{aligned} H^{(3)}_{lc,1} = &-\frac{1}{k\eta^2}\Big\langle\left((N-1)\tilde{N}' + (\tilde{N}-1)N' - \frac{1}{2}(N+\tilde{N}-2)(N'+\tilde{N}')\right)\Big\rangle - \\ &-\Big\langle\frac{1}{2\pi k}\int_0^l d\sigma\,W'_L\bar{W}_LW'_R\bar{W}_R + \frac{1}{2k\eta}\left(\int_0^l\frac{d\sigma}{\pi}\,\mathrm{Im}(\bar{W}_LW'_R)\right)^2\Big\rangle, \end{aligned} \qquad (3.34)$$

where we used $l = 2\pi\eta$. The term in the first line vanishes due to condition

$$N = \tilde{N}\,. \qquad (3.35)$$

The first term in the second line completely cancels (3.21), while the second term in this line cancels the second term in (3.18). Hence the the lightcone spectrum up to next to leading order in $1/k$ is

$$H_{lc} = H_{lc,0} + \sum_{i=1}^{3} H^{(i)}_{lc,1} = 2 + N' + \tilde{N}' - \frac{2(N+\tilde{N}-2)}{p_-} - \frac{2(N+\tilde{N}-2)(N'+\tilde{N}')}{k(p_-)^2}\,. \qquad (3.36)$$

This can also be written as

$$H_{lc} = 2 + N' + \tilde{N}' - \frac{4(N-1)}{p_-} - \frac{4(N-1)(N'+\tilde{N}')}{k(p_-)^2} \qquad (3.37)$$

due to the physical state condition. This however is not the end of the story, as one must remember that the original space contains a compact direction ψ which implies quantization of the SU(2) spin. Corresponding Killing vector is

$$-i\partial_\psi = \frac{-i\partial_{X^+} - k(-i\partial_{X^-})}{2}\,. \qquad (3.38)$$

The momentum which generates translations in ψ has to be quantized, which implies

$$\frac{p_+ - kp_-}{2} = -\frac{kp_- + H_{lc}}{2} \in Z\,. \tag{3.39}$$

Substituting (3.36) and retaining terms up to next to leading order recasts this constraint as

$$-\frac{kp_-}{2} - 1 - \frac{N' + \tilde{N}'}{2} + \frac{2(N-1)}{p_-} = -i\partial_\psi \in Z\,. \tag{3.40}$$

4. $AdS_3 \times S^3$: exact quantization

Let us study states with $j, h \sim \mathcal{O}(k)$ in the large k limit. In the following we will keep terms up to the second order in the expansion in powers of $1/k$. The physical state condition for states with oscillator number N

$$-\frac{h(h-1)}{k} + \frac{j(j+1)}{k} + N = 1\,, \tag{4.1}$$

relates h and j as

$$h = j + 1 + \frac{(N-1)k}{2j} - \frac{1}{k}\frac{(N-1)k}{2j}\left(\frac{(N-1)k}{2j} + 1\right). \tag{4.2}$$

We will generate all states from the primary state $|j\rangle$ which has the SL(2) spin h related to the SU(2) spin j as in (4.2). The quantum numbers of this state are given by

$$m^{\mathrm{SL}} = \tilde{m}^{\mathrm{SL}} = h\,, \qquad m^{\mathrm{SU}} = \tilde{m}^{\mathrm{SU}} = j\,. \tag{4.3}$$

These quantum numbers are related to η and H_{lc}, which appear in the Penrose limit, as follows

$$H_{lc} = m^{\mathrm{SL}} + \tilde{m}^{\mathrm{SL}} - m^{\mathrm{SU}} - \tilde{m}^{\mathrm{SU}}\,, \tag{4.4}$$

$$2\eta = -p_- = \frac{m^{\mathrm{SL}} + \tilde{m}^{\mathrm{SL}} + m^{\mathrm{SU}} + \tilde{m}^{\mathrm{SU}}}{k}\,. \tag{4.5}$$

It was argued in [1] that bulk isometries correspond to string zero modes in pp wave background. In the present context this implies that the action of the zero modes of the SU(2) current algebra $J_0^{-,\mathrm{SU}}, \tilde{J}_0^{-,\mathrm{SU}}$ on the state $|j\rangle$ corresponds to the action of zero mode creation operators $a_0^\dagger, \tilde{a}_0^\dagger$ on the lightcone vacuum in the pp wave background. In other words, we have the correspondence

$$a_0^\dagger \leftrightarrow J_0^{-,\mathrm{SU}}\,, \qquad a_0 \leftrightarrow J_0^{+,\mathrm{SU}}\,, \tag{4.6}$$

$$\tilde{a}_0^\dagger \leftrightarrow \tilde{J}_0^{-,\mathrm{SU}}\,, \qquad \tilde{a}_0 \leftrightarrow \tilde{J}_0^{+,\mathrm{SU}}\,. \tag{4.7}$$

This correspondence extends to the nonzero modes

$$a_{n+}^\dagger \leftrightarrow J_{-n}^{-,\mathrm{SU}}\,, \qquad a_{n+} \leftrightarrow J_n^{+,\mathrm{SU}}\,, \tag{4.8}$$

$$a_{n-}^\dagger \leftrightarrow J_{-n}^{+,\mathrm{SU}}\,, \qquad a_{n+} \leftrightarrow J_n^{-,\mathrm{SU}}\,, \tag{4.9}$$

$$\tilde{a}_{n+}^\dagger \leftrightarrow \tilde{J}_{-n}^{+,\mathrm{SU}}\,, \qquad \tilde{a}_{n+} \leftrightarrow \tilde{J}_n^{-,\mathrm{SU}}\,, \tag{4.10}$$

$$\tilde{a}_{n-}^\dagger \leftrightarrow \tilde{J}_{-n}^{-,\mathrm{SU}}\,, \qquad \tilde{a}_{n-}^\dagger \leftrightarrow \tilde{J}_n^{+,\mathrm{SU}}\,, \tag{4.11}$$

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with similar correspondence for the AdS_3 part. It is easy to see that this identification implies correct matching for the transformation properties under the rotation in the Z plane generated by $J_0^{3,\mathrm{SU}} - \tilde{J}_0^{3,\mathrm{SU}}$. Furthermore, the physical state condition (2.53) is naturally interpreted as level matching on the $AdS_3 \times S^3 \times \mathcal{M}$ side. Now we would like to show that the lightcone energy of the physical states precisely reproduces the result obtained from lightcone quantization (3.36), up to next to leading order in $1/k$ expansion. Let us first note that

$$m^{\mathrm{SL}} = \tilde{m}^{\mathrm{SL}} = h\,, \qquad m^{\mathrm{SU}} = j - N'\,, \qquad \tilde{m}^{\mathrm{SU}} = j - \tilde{N}'\,. \tag{4.12}$$

Using this relation, together with (4.4) and (4.2) we obtain

$$H_{lc} = 2 + N' + \tilde{N}' + \frac{2(N-1)k}{2j} - \frac{2(N-1)}{k}\left(\frac{k}{2j}\right)^2\left(\frac{(N-1)k}{2j} + 1\right). \tag{4.13}$$

One can also write down the relation between p_- and j using (4.5) and (4.2):

$$-p_- = \frac{1}{k}\left[2 + 4j + \frac{2(N-1)k}{2j} - N' - \tilde{N}'\right], \tag{4.14}$$

which implies

$$2j = -\frac{kp_-}{2} - 1 + \frac{2(N-1)}{p_-} + \frac{N' + \tilde{N}'}{2} \tag{4.15}$$

up to terms which are $\mathcal{O}(1/k)$. Note that the requirement for $2j$ to be an integer reproduces precisely the constraint (3.40) which appeared due to the compactness of the space. In fact, what should appear in the right hand side of (3.40), is

$$-i\partial_\psi = 2j - (N' + \tilde{N}')\,, \tag{4.16}$$

meaning that (3.40) and (4.15) are completely equivalent. Substituting (4.15) into (4.13) reproduces the lightcone hamiltonian (3.36).

5. Discussion

In this paper we studied bosonic string propagating in the Penrose limit of $AdS_3 \times S^3 \times \mathcal{M}$. We identified states in the leading order in $1/k$ expansion (that is, states in the pp wave background) with physical states in $AdS_3 \times S^3 \times \mathcal{M}$. When one goes beyond the leading order, the pp wave background receives corrections. The worldsheet theory of a lightcone string propagating there becomes interacting, with the appearance of quartic terms in the action. We computed next to leading order corrections to the spectrum in perturbation theory and found precise agreement with the results from exact quantization in original $AdS_3 \times S^3 \times \mathcal{M}$ background. The correspondence has been worked out for states in the unitary representations with AdS_3 spin $1/2 < h < (k+1)/2$. The incorporation of spectrally flowed states should be relatively simple. One should also be able to extend the results of this paper to superstring case. Let us mention some other interesting problems and directions for future work. We would like to reiterate that $AdS_3 \times S^3$ background, because of its solvability, seems to be a good toy model for studying various features of the Penrose limit in string theory. For example, one may be able to see directly how correlation functions behave under the Penrose limit both in the bulk and in the boundary theory. Perhaps one can also make some progress in constructing a holographic dual to the pp wave background.

One of the most interesting questions raised by BMN construction is why operators that correspond to the excited string states take the form of single trace operators with phase insertions [1]. It is possible that $AdS_3 \times S^3$ example will be useful in understanding this better. However it is likely that pursuing this question will require generalization of the result of this paper to the case of nonzero RR flux, in order to be able to study a nonsingular boundary theory. In fact, it is not hard to construct operators in the boundary theory that correspond to states that are created by the action of zero mode oscillators on the lightcone vacuum in pp wave background. The reason is that these states essentially correspond to chiral primary observables in the boundary SCFT which are independent of moduli. Corresponding worldsheet operators have been constructed for the theory with NS-NS flux in [28] and they agree with the spectrum of chiral primaries in $(T^4)^{kp}/S_{kp}$ orbifold [29], even though the two theories correspond to different points in the moduli space of D1/D5 system.

At any rate, it seems like generalization of our results to $AdS_3 \times S^3$ background with RR flux deserves further attention. In this case the exact quantization scheme is available [30]. Another interesting direction would be studying other backgrounds with RR flux, where the exact quantization procedure is lacking. Note that we worked out first two leading terms in the $1/k$ expansion, but in principle one can go further and extend the results to higher orders in perturbation theory. This procedure would provide an alternative way of quantizing string theory in $AdS_3 \times S^3 \times \mathcal{M}$ and may prove very useful when applied to backgrounds that cannot be quantized exactly. Even in AdS_3 we may learn something new, as the conventional quantization procedure deals with the euclidean version of AdS_3, and the results of analytic continuation are not completely understood. For example, one may try to understand better D-branes in lorentzian AdS_3 and make a connection with algebraic results of [31]–[34]. D-branes in pp wave backgrounds have been investigated recently in [35]–[39].

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References

[1] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[2] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[3] R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave ramond- ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[4] J.G. Russo and A.A. Tseytlin, *On solvable models of type IIB superstring in NS-NS and R-R plane wave backgrounds*, *J. High Energy Phys.* **04** (2002) 021 [hep-th/0202179].

[5] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[6] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[7] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[8] C.G. Callan and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys.* **B 472** (1996) 591 [hep-th/9602043].

[9] G.T. Horowitz, J.M. Maldacena and A. Strominger, *Nonextremal black hole microstates and U-duality*, *Phys. Lett.* **B 383** (1996) 151 [hep-th/9603109].

[10] J.M. Maldacena and A. Strominger, *AdS_3 black holes and a stringy exclusion principle*, *J. High Energy Phys.* **12** (1998) 005 [hep-th/9804085].

[11] C.G. Callan, J.A. Harvey and A. Strominger, *Worldbrane actions for string solitons*, *Nucl. Phys.* **B 367** (1991) 60.

[12] A. Dabholkar, G. Gibbons, J. Harvey, F. Ruiz-Ruiz, *Superstrings and solitons*, *Nucl. Phys.* **B 340** (1990) 33.

[13] A.A. Tseytlin, *Extreme dyonic black holes in string theory*, *Mod. Phys. Lett.* **A 11** (1996) 689 [hep-th/9601177].

[14] A. Giveon, D. Kutasov and N. Seiberg, *Comments on string theory on AdS_3*, *Adv. Theor. Math. Phys.* **2** (1998) 733 [hep-th/9806194].

[15] D. Kutasov and N. Seiberg, *More comments on string theory on AdS_3*, *J. High Energy Phys.* **04** (1999) 008 [hep-th/9903219].

[16] J. de Boer, H. Ooguri, H. Robins and J. Tannenhauser, *String theory on AdS_3*, *J. High Energy Phys.* **12** (1998) 026 [hep-th/9812046].

[17] A. Giveon and D. Kutasov, *Notes on AdS_3*, *Nucl. Phys.* **B 621** (2002) 303 [hep-th/0106004].

[18] R. Argurio, A. Giveon and A. Shomer, *Superstrings on AdS_3 and symmetric products*, *J. High Energy Phys.* **12** (2000) 003 [hep-th/0009242].

[19] N. Seiberg and E. Witten, *The D1-D5 system and singular CFT*, *J. High Energy Phys.* **04** (1999) 017 [hep-th/9903224].

[20] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, hep-th/0204051.

[21] S. Frolov and A.A. Tseytlin, *Semiclassical quantization of rotating superstring in $AdS_5 \times S^5$*, *J. High Energy Phys.* **06** (2002) 007 [hep-th/0204226].

[22] E. Kiritsis and B. Pioline, *Strings in homogeneous gravitational waves and null holography*, hep-th/0204004.

[23] R. Penrose, *Any spacetime has a plane wave as a limit*, in *Differential geometry and relativity*, Reidel, Dordrecht, 1976, pp. 271-275

[24] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. Quant. Grav.* **19** (2002) L87–L95 [hep-th/0201081].

[25] J. Polchinski, *String theory* vol. 1: *An introduction to the bosonic string*, Cambridge, UK: Univ. Press 1998.

[26] J.M. Maldacena and H. Ooguri, *Strings in AdS_3 and SL(2,R) WZW model 1*, *J. Math. Phys.* **42** (2001) 2929 [hep-th/0001053].

[27] L.J. Dixon, D. Friedan, E.J. Martinec, S.H. Shenker, *The conformal field theory of orbifolds*, *Nucl. Phys.* **B 282** (1987) 13.

[28] D. Kutasov, F. Larsen and R.G. Leigh, *String theory in magnetic monopole backgrounds*, *Nucl. Phys.* **B 550** (1999) 183 [hep-th/9812027].

[29] F. Larsen and E.J. Martinec, *U(1) charges and moduli in the D1-D5 system*, *J. High Energy Phys.* **06** (1999) 019 [hep-th/9905064];

[30] N. Berkovits, C. Vafa and E. Witten, *Conformal field theory of AdS background with Ramond-Ramond flux*, *J. High Energy Phys.* **03** (1999) 018 [hep-th/9902098].

[31] A. Giveon, D. Kutasov and A. Schwimmer, *Comments on D-branes in AdS_3*, *Nucl. Phys.* **B 615** (2001) 133 [hep-th/0106005].

[32] A. Parnachev and D.A. Sahakyan, *Some remarks on D-branes in AdS_3*, *J. High Energy Phys.* **10** (2001) 022 [hep-th/0109150].

[33] B. Ponsot, V. Schomerus and J. Teschner, *Branes in the euclidean AdS_3*, *J. High Energy Phys.* **02** (2002) 016 [hep-th/0112198].

[34] P. Lee, H. Ooguri and J.-W. Park, *Boundary states for AdS_2 branes in AdS_3*, *Nucl. Phys.* **B 632** (2002) 283 [hep-th/0112188].

[35] A. Dabholkar and S. Parvizi, *Dp-branes in pp-wave background*, hep-th/0203231.

[36] A. Kumar, R.R. Nayak and Sanjay, *D-brane solutions in pp-wave background*, hep-th/0204025.

[37] D.-S. Bak, *Supersymmetric branes in pp wave background*, hep-th/0204033.

[38] K. Skenderis and M. Taylor, *Branes in AdS and pp-wave spacetimes*, hep-th/0204054.

[39] M. Billó and I. Pesando, *Boundary states for GS superstrings in an Hpp background*, *Phys. Lett.* **B 536** (2002) 121 [hep-th/0203028].

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